The Overshoot Problem and Giant Structures

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Abstract

Models of small-field inflation often suffer from the overshoot problem. A particularly efficient resolution to the problem was proposed recently in the context of string theory. We show that this resolution predicts the existence of giant spherically symmetric overdense regions with radius of at least 110 Mpc. We argue that if such structures will be found they could offer an experimental window into string theory.

1 Introduction

Recent observations strongly support a period of inflation in the early universe [1]. In fact many models of inflation, some of which were quite popular not too long ago, are now ruled out [2]. In light of this remarkable progress it is tempting to ask, perhaps prematurely, what might be the experimental signatures of a pre-inflationary period, assuming such a period existed. This and related questions were addressed by various authors (see e.g. [3]). Our goal here is to connect this question with the overshoot problem associated with small-field inflation.

If the total number of e-foldings during inflation, N_e^{tot}, is much larger then the number of e-foldings needed to resolve the big-bang puzzles, N_e^{BB}, then the probability of finding any pre-inflationary signature in the visible universe is likely to be negligible. The difficulties encountered in realizing in string theory models of inflation with a large number of e-foldings perhaps should be viewed as an indication that N_e^{tot} is not much larger than N_e^{BB}, in which case pre-inflationary signatures might be detectable.

Most stringy models of inflation suggested so far are models of small-field inflation. Such models often suffer from the overshoot problem [4], which in this context means that, against the spirit of inflation, one needs to tune the initial conditions of the inflaton for inflation to take place. This problem received a lot of attention over the years (mostly in the context of moduli stabilization), and various ways to overcome it were proposed (see e.g. [5]). A nice aspect of the overshoot problem associated with small-field inflation is that any mechanism which resolves it should act mostly right before inflation, and so it is possible that it leaves detectable imprints (assuming $\Delta N_e = N_e^{tot} - N_e^{BB}$ is not too large).

Recently a particularly efficient resolution to the problem was suggested in [6]. The resolution relies on the existence of particles with mass that depends on the expectation value of the inflaton. These particles push the inflaton in the opposite direction than the static potential does ($m_{,\phi} V_{,\phi} < 0$), and slow down the inflaton at the slow roll region. In the present paper we show that this pre-inflationary scenario has a distinct signature: a formation of giant spherically symmetric overdense regions with a radius of at least 110 Mpc. Each of these particles present at the beginning of inflation provides the seed of a single giant spherically symmetric structure. The properties of these giant structures are fixed by $m_{,\phi}$.

The paper is organized as follows: In the next section we review the mechanism proposed in [6] to resolve the overshoot problem. In section 3 we show that this mechanism leads to the formation of overdense regions and study their properties. We argue that if N_e^{tot} is not much larger than N_e^{BB} some of these giant structures should be found in the visible universe.

2 A review of [6]

Most models of inflation in string theory are small-field models[1] (for reviews of stringy inflation see [9]). Namely, the slow roll conditions are satisfied over a small distance in the field space of the canonically normalized inflaton. Roughly speaking the reason is that in string theory there are severe constraints on the possible terms that could contribute to the inflaton potential. Hence it is hard to construct a potential for the inflaton that satisfies the slow roll condition over a large distance in field space.

Typically models of small-field inflation suffer from the overshoot problem. This problem was first raised in the context of moduli stabilization in [4] and is very much relevant also for small-field inflation (see e.g [10]). The problem is that a generic initial condition is not at the region where the slow roll conditions are met, and if we start away from the slow roll region the inflaton will overshoot it without ever being dominated by the potential energy. This happens because in small-field inflation, by definition, the slow roll region is small and the Hubble friction does not have enough time to slow down the inflaton. As a result the universe does not inflate despite the fact that the inflaton passes through the slow roll region.

Most of the resolutions proposed to the overshoot problem [5] are based on the fact that the energy density of a scalar field dominated by its kinetic energy scales like $1/a(t)^6$. Thus, almost any other contribution to the energy density, like matter or radiation, does not drop as fast and eventually it takes over. As a result the Hubble friction becomes larger and it could potentially slow down the inflaton at the slow roll region. Typically, however, these kind of mechanisms are more efficient when addressing the overshoot problem in the context of moduli stabilization than in the context of small-field inflation.

Recently [6] a more efficient resolution to the problem was proposed in the context of

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[1]As far as we know the only large-field model of inflation in string theory was proposed recently [8].

string theory[2]. It was noticed in [6] that at least in some cases there are particles with masses that depend on the inflaton and satisfy

$$m_{,\phi} \; V_{,\phi} < 0. \tag{2.1}$$

Such particles will change dramatically the dynamics of the inflaton in the following way. Much like any other particles these particles are expected to be produced thermally when the energy density is high (Fig. 1(a)). Since $m_{,\phi} \; V_{,\phi} < 0$ their density will induce an effective potential that pushes the inflaton in the opposite direction than the static potential does (Fig. 1(a)). As the universe expands the density of these particles and their effective potential grows weak and the inflaton minimizes the time dependent potential energy (Fig. 1(b)). Upon entering the slow roll region the slope of the static potential becomes negligible and is not able to balance the potential induced by the particles (Fig. 1(c)). Hence the particles get diluted while the inflaton stays in the slow roll region (Fig. 1(d)). This sets up the initial condition for the inflaton at the slow roll region.

This mechanism is rather general and is expected to work in any model that includes particles that satisfy $m_{,\phi} \; V_{,\phi} < 0$ with a large enough $m_{,\phi}$. Let us review how this comes about in the stringy example considered in [6]. The setup is of modular inflation [11]. We consider ten dimensional string theory with topology $R^{3,1} \times X$ where X is a six dimensional compact manifold. We *assume* that all moduli but the volume of X are fixed and consider the possibility that the inflaton is related to the volume of X. The relation between the canonically normalized inflaton and V_X is

$$L = e^{\phi/\sqrt{24}}, \quad \text{where} \quad L \sim V_X^{1/6}. \tag{2.2}$$

String theory allows only for a small number of terms to appear in the classical potential for L (or ϕ). These are due to wrapped branes, fluxes and curvature. For example in type IIB we can have

$$V(\phi) = \sum_a C_a L^{-a}, \quad \text{with} \quad a = 8, 10, 12, 16, \tag{2.3}$$

and in type IIA

$$V(\phi) = \sum_a C_a L^{-a}, \quad \text{with} \quad a = 8, 9, 10, 11, 14, 18. \tag{2.4}$$

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[2]In relation with the overshoot problem associated with moduli stabilization a similar mechanism was proposed in [7].
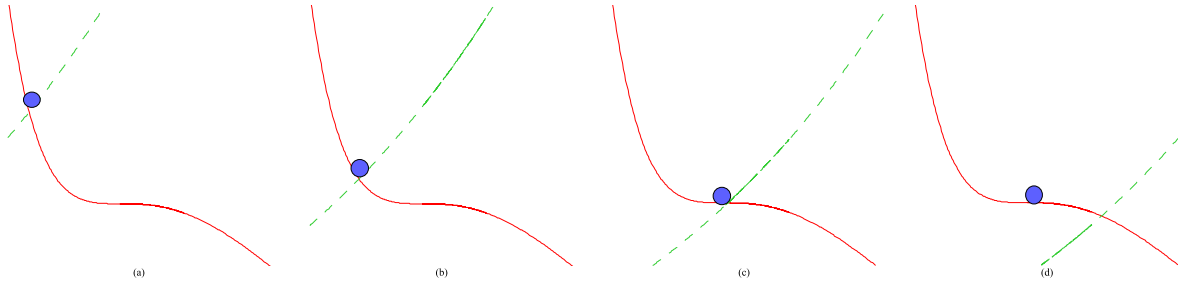
Figure 1: A heuristic illustration of how a time dependent potential, that scales like $1/a^3(t)$, resolves the overshoot problem of small-field models of inflation. The solid line represents the static potential, the dashed line the time dependent potential that is induced by the particles and the blue dot the time dependent value of the inflaton.

Some of the constants, C_a, can be negative but most of them cannot, and depending on the topology of X some of them have to vanish. Moreover all the terms go to zero at least as fast as $1/L^8$ when $L \to \infty$. As a result it is practically impossible to satisfy the slow roll condition over a wide region of ϕ. Namely, in this setup one cannot construct a large-field model of inflation. In fact it is not easy to construct a model of small-field inflation either. The simplest model is of an inflection point inflation constructed with the help of three terms. As argued above and as was illustrated in [6] this model suffers from the overshoot problem. It can be shown that the overshoot problem is generic in this setup.

The nice feature of the model is that it includes also particles with $m_{,\phi} \, V_{,\phi} < 0$ which resolve the overshoot problem via the mechanism described above. The relevant particles are D-branes which wrap some of the cycles of X. The masses of these particles are

$$M_{D-brane} = c_B L^B, \quad \text{with} \quad B > 0, \tag{2.5}$$

where c_B is a positive constant that depends on the value of the other moduli field (in particular the dilaton) which are assumed to be fixed. Since no concrete mechanism for fixing them was proposed in [6] c_B cannot be calculated in this approximated setup. Again the possible values of B depend on the type of string theory under consideration, and the topology of X. In type IIA the possible values of B are 1 or 3 and in type IIB there is only one possible value $B = 2$.

A natural question to ask is whether these particles lead to any clear experimental prediction which could test the mechanism of [6]. Much like monopoles, or any other

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heavy particles, they will get diluted exponentially fast during inflation, and so their contribution to the total energy in the universe at the end of inflation is negligible. On the other hand their imprint on structure formation could be significant and perhaps even detectable. This is the subject of the next section.

3   Giant Structures

The discussion in the previous section was done in the homogenous approximation. Namely, we considered the net effect particles with $m_{,\phi}V_{,\phi} < 0$ have on the evolution of the zero mode of the inflaton. During inflation the proper distance between the particles grows exponentially fast and very quickly the homogenous approximation breaks down at macroscopic scales. Since these particles couple *directly* to the inflaton and since this coupling played such an important role in the dynamics of the inflaton we expect the inhomogeneities due to the individual particles to induce inhomogeneities of the inflaton.[3] It is well known that inhomogeneities of the inflaton provide the seeds of structure formation. Hence, it is reasonable to suspect that these particles could affect structure formation in an interesting way.

We wish to study the effect a single particle with $m_{,\phi}V_{,\phi} < 0$ present at the beginning of inflation has on structure formation. Before we turn to the actual calculation it is instructive to recall the intuitive relation between the inflaton and structure formation [12]. During inflation ϕ plays the role of a clock and in particular it determines the time at which inflation ends. A non-uniform inflaton will cause inflation to end in a non-uniform fashion. Regions where inflation ends first had more time to expand after inflation and so their density is slightly smaller. A particle with $m_{,\phi}V_{,\phi} < 0$ pushes the inflaton in the opposite direction than the static potential does. Hence inflation will end first away from the particle. Therefore, we expect such a particle to provide the seed of an overdense region.

To estimate the size and density of this overdense region we start by calculating the non-uniform shape of the inflaton caused by a single particle with $m_{,\phi}V_{,\phi} < 0$. For simplicity we consider a spatially flat universe

$$ds^2 = -dt^2 + a(t)^2 dx_i^2.$$

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[3]On top of the usual inflaton's inhomogeneities due to quantum effects.

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Since the mass of the particle depends on the inflaton its presence modifies the inflaton equation of motion directly

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a(t)^2}\nabla^2\phi + V_{,\phi} + m_{,\phi}\frac{\delta^3(x_i)}{a(t)^3} = 0. \tag{3.1}$$

To solve this equation we separate the solution into two parts: the zero mode and the r dependent perturbation

$$\phi(r,t) = \phi(t) + \delta\phi(r,t). \tag{3.2}$$

$\phi(t)$ solves the standard equation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0, \tag{3.3}$$

and $\delta\phi(r,t)$ solves the linear equation

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{1}{a(t)^2}\nabla^2\delta\phi + V_{,\phi\phi}\delta\phi + \frac{m_{,\phi}}{a(t)^3}\delta^3(x_i) = 0. \tag{3.4}$$

During the period of slow roll inflation the $V_{,\phi\phi}$ term can be neglected and we have

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{1}{a(t)^2}\nabla^2\delta\phi + \frac{m_{,\phi}}{a(t)^3}\delta^3(x_i) = 0. \tag{3.5}$$

As expected we ended up with the familiar equation for the perturbation only that now there is a source located at the origin.

At early times, when $ra(t) \ll 1/H$, the time derivatives in (3.5) are negligible and the solution takes the familiar form

$$\delta\phi = -\frac{m_{,\phi}}{4\pi ra(t)}, \tag{3.6}$$

which in momentum space gives

$$\delta\phi_k = -\frac{m_{,\phi}}{(2\pi)^{3/2}k^2 a(t)}. \tag{3.7}$$

At late times the Hubble friction term in (3.5) becomes important and cause a freeze out of different modes at different times. As usual in an accelerating universe the freeze out occurs when $a(t) \sim k/H$ and so the late time solution reads

$$\delta\phi_k = -C\frac{Hm_{,\phi}}{k^3}, \tag{3.8}$$

where C is a positive constant. In the appendix we calculate C and find

$$C = \frac{1}{\sqrt{32\pi}}. \tag{3.9}$$

Next we use the well know relation (see e.g. [13]) between the inflaton and structure formation

$$\delta_k = \frac{2}{5}\frac{k^2}{H_0^2}T(k)R_k. \tag{3.10}$$

Here δ_k is the momentum mode of $\delta\rho/\rho$, H_0 is the present Hubble scale, R_k is determined during inflation

$$R_k = -\frac{H_{inflation}}{\dot{\phi}}\delta\phi_k, \tag{3.11}$$

and $T(k)$ is the transfer function.

For simplicity we make two approximations. First we take the most naive approximation to the transfer function

$$T(k) = \begin{cases} 1 & \text{for } k < k_{eq} \\ k_{eq}^2/k^2 & \text{for } k > k_{eq}, \end{cases} \tag{3.12}$$

where $1/k_{eq}$ is the comoving Hubble length at matter-radiation equality $\sim 14\Omega_M^{-1}h^{-2} \sim$ 110 Mpc. This approximation captures, in a crude fashion, the fact that the formation of structure during the radiation dominated era is suppressed compered to the matter dominated era. However it neglects the less clean physics associated with modes which enter the horizon during the radiation dominated era. The corrections to (3.12) are logarithmic in k (see e.g. [13]).

The second approximation is the fact that we ignore the effects due to dark energy which become important at late times. That is we take a trivial growth function. If the size of the formed structure is not too large a significant portion of the evolution is in the matter dominated era, and the approximation is expected to be fairly good.

Clearly these approximations are rather crude, and to have a better description of the formed structure we need to go beyond them. This will be done elsewhere. Here we take advantage of the fact that these approximations yield simple expressions that, we believe, capture the main effects.

Let us first assume that the modes that are most relevant for the structure formation are modes with $k < k_{eq}$. Momentarily we will see what is the condition $m_{,\phi}$ has to satisfy for this to be the case. In this case $T(k) = 1$, and

$$\delta_k = -\frac{2}{5}\frac{k^2}{H_0^2}\frac{H_{inflation}}{\dot{\phi}}\delta\phi_k. \tag{3.13}$$

Using the slow roll equation ($3H\dot{\phi} = -V_{,\phi}$) and (3.8, 3.9) we get

$$\delta_k = -\frac{V^{3/2}m_{,\phi}}{10\sqrt{6\pi}H_0^2 V_{,\phi}k} = 1.2 \times 10^{-5}\frac{|m_{,\phi}|}{H_0^2 k}, \tag{3.14}$$

where in the second equality we used the COBE normalization ($\frac{V^{3/2}}{V_{,\phi}} = 5.2 \times 10^{-4}$) and (2.1).

Transforming back into position space we find that

$$\frac{\delta\rho}{\rho} \sim 10^{-5}\frac{|m_{,\phi}|}{H_0^2 r^2}, \tag{3.15}$$

and that the size of the overdense region (fixed by $\delta\rho = \rho$) is

$$r \sim m_{,\phi}^{1/2}13 \text{ Mpc}. \tag{3.16}$$

We see that for the assumption that the most relevant modes are modes with $k < k_{eq}$ to hold $m_{,\phi}$ should be larger than $\sim (110/13)^2 \sim 70$, which is quite large given that $m_{,\phi}$ is a dimensionless parameter.

Next we consider the probably more realistic case with $m_{,\phi} < 70$ in which the relevant modes have $k > k_{eq}$. Then $T(k) = k_{eq}^2/k^2$ and

$$\delta_k = -\frac{2}{5}\frac{k_{eq}^2}{H_0^2}\frac{H_{inflation}}{\dot{\phi}}\delta\phi_k. \tag{3.17}$$

Following the same steps as before we get

$$\delta_k = 1.2 \times 10^{-5}\frac{k_{eq}^2|m_{,\phi}|}{H_0^2 k^3} = 1.6 \times 10^{-2}\frac{|m_{,\phi}|}{k^3}. \tag{3.18}$$

Transforming back into position space we find for $r < k_{eq}^{-1} \sim 110$ Mpc that $\frac{\delta\rho}{\rho}$ is approximately a constant[4]

$$\frac{\delta\rho}{\rho} \sim 10^{-2}|m_{,\phi}|. \tag{3.19}$$

Therefore, for $m_{,\phi}$ of order 1 we expect $\frac{\delta\rho}{\rho}$ to be fairly small $\sim 10^{-2}$. However, this small effect takes place within a giant spherically symmetric region with a radius of about 110 Mpc. Within this region we expect to find the usual structures (due to quantum fluctuations during inflation) at much smaller scales but with $\frac{\delta\rho}{\rho}$ much larger than $10^{-2}|m_{,\phi}|$.

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[4]Neglecting logarithmic effects which are ignored since the transfer function we work with, (3.12), neglects various logarithmic corrections.

Put differently each particle creates a giant region of size of order 220 Mpc in which the universe looks roughly like it used to look at $z \sim m_{,\phi}/100$. That is, it has similar structure as in the rest of the universe: Most of the galaxies are in a nearly spherically symmetric overdense region of size of a few Mpc. Some of these almost spherically symmetric regions are connected via filaments with smaller density and size of order 10-30 Mpc. The filaments are connect via walls with an even lower density, and there are voids between the walls. The only difference is that the average density is larger than in the rest of the universe.

Natural questions to ask at this stage are:

1. How large is $m_{,\phi}$ in the scenario of [6]?

2. How many such overdense regions should we expect to find in the visible universe?

Let us begin with the first question. As described in the previous section quite generically we expect $m_{,\phi}$ to be fairly large in order to resolve the overshoot problem. From (2.2, 2.5) we find that in the concrete example studied in [6]

$$m_{,\phi} = \frac{c_B \, B}{\sqrt{24}} L_{SR}^B. \tag{3.20}$$

In [6] it was shown that the COBE normalization implies

$$L_{SR} \cong 50 a_3^{1/8}, \tag{3.21}$$

where a_3 is a constant that, much like c_B, depends on the values at which the other moduli are fixed and so it is not known within the approximated setup of [6]. Combining (3.20) with (3.21) we get

$$m_{,\phi} \cong c_B B a_3^{B/8} \frac{50^B}{\sqrt{24}}. \tag{3.22}$$

We see that unless c_B and a_3 obtain unusually small values $m_{,\phi}$ is larger then 1. Hence the effect such a particle has on structure formation should be significant.

It is less clear whether $m_{,\phi}$ is smaller or larger than ~ 70. Again with the assumption that c_B and a_3 are of order one we see that for $B = 1$ it is most likely that $m_{,\phi} < 70$ while for $B = 3$ we probably have $m_{,\phi} > 70$. The type IIB case ($B = 2$) tends to give $m_{,\phi} > 70$, but it could easily give $m_{,\phi} < 70$. Thus we do not know if such a particle will lead to the formation of a structure larger than 110 Mpc or not. We reemphasis that in models in which all modui but the inflaton are fixed we do expect to be able to compute $m_{,\phi}$ and to make sharp prediction about the giant structure.

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We assumed so far that the particles are isolated. However since they interact also via a scalar (the inflaton) exchange it is possible that they form clusters in the pre-inflationary stage.[5] In [18] this was shown to happen in the mass varying neutrinos scenario of [19]. Such a cluster of particles will have $m_{,\phi}^{eff} = N_{cluster} \, m_{,\phi}$. Thus this could drastically improve the local efficiency of the mechanism of [6] especially when $m_{,\phi}$ is small (which is likely to be the case in an effective field theory set-up, since $m_{,\phi}$ is dimensionless) and could change the properties of the giant structure. This interesting possibility will be studied elsewhere.

Unfortunately, even in models in which all moduli but the inflaton are fixed, the answer to the second question is not clear. The reason is that the answer is exponentially sensitive to N_e^{tot}: The density of these particles at the beginning of inflation, n_0, must be large enough to slow down the inflaton at the slow roll region. A rough lower bound on n_0 comes from demanding that just before infaltion begins the slope of the potential induced by the particles is larger then the one of the static potential. This gives

$$n_0 > \frac{V_{SL}}{m_{,\phi}}. \tag{3.23}$$

Using the COBE normalization[6] we find that the total number of particles within the Horizon at the beginning of inflation is

$$N_0 = \frac{n_0}{H^3} > \frac{1}{m_{,\phi}H} \sim \frac{10^4}{m_{,\phi}\sqrt{\epsilon}}. \tag{3.24}$$

Typically in small-field inflation $\epsilon \ll \eta \sim 0.05$. Thus a fair estimate is $N_0 \sim 10^6$. This implies that the number of giant overdense regions in the visible universe is about

$$10^6 e^{-3(N_e^{tot} - N_e^{BB})}, \tag{3.25}$$

and that for the number of giant overdense regions in the universe to be of order 1 we should have

$$\Delta N_e = N_e^{tot} - N_e^{BB} \sim 5. \tag{3.26}$$

With our present knowledge it is hard to tell whether this is likely or not to be the case. An argument against this is that $\Delta N_e/N_e^{BB} \sim 1/10$. So there is an extra tuning in the

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[5]We thank the referee of the paper for raising this issue.

[6]Note that the COBE normalization fixes $V^{3/2}/V_{,\phi}$ at around N_e^{BB} e-foldings before the end of inflation, while here we are using it N_e^{tot} e-foldings before the end of inflation. The difference between the two is not necessarily negligible. However, since we are merely trying to estimate N_0 this is a reasonable approximation to use.

model. On the other hand one can argue that since the amount of fine tuning needed in order to have $N_e^{tot} \gg 1$ grows with N_e^{tot} it is likely that N_e^{tot} is not much larger then N_e^{BB}.

We emphasis that (3.26) is merely an estimate based on the assumption of high scale inflation. In low scale inflation with V_{SL} as low as TeV^4 [16] we find that for the number of giant overdense regions in the universe to be of order 1 we should have

$$\Delta N_e \sim 10, \tag{3.27}$$

and, since in low scale inflation $N_e^{BB} \sim 40$, that $\Delta N_e / N_e^{BB} \sim 1/4$.

Note that there are other particles in the model with mass that depends on the inflaton, but with $m_{,\phi} \, V_{,\phi} > 0$. These are the perturbative excitations with

$$m_{per} \sim L^{-A}, \quad A > 0, \tag{3.28}$$

which are also expected to be produce thermally at stage (a) of Fig. 1. Hence some of them are expected to be found at the begging of inflation. Should we conclude from this that if giant overdense regions are found then giant voids should be found as well? At least in the model of [6] the answer is no. The reason is that since from (3.21) we expect $L_{SR} \gg 1$ we find from (3.28) that for the perturbative excitation

$$|m_{,\phi}^{per}| \ll 1. \tag{3.29}$$

Hence the effect of these particles on structure formation is expected to be negligible (even when ΔN_e is small). In fact, this condition must be satisfied. Otherwise the mechanism described in section 2 will not resolve the overshoot problem as the induced potential due to the particles with $m_{,\phi} \, V_{,\phi} > 0$ will cancel the induced potential due to the particles with $m_{,\phi} \, V_{,\phi} < 0$.

This of course does not mean that there are no other models in which giant voids could be formed via the mechanism described here (with $m_{,\phi} \, V_{,\phi} > 0$). We mention this since, in relation with the WMAP cold spot [20], some arguments were already made for a giant void [21]. See however [22].

4   Summary

In this paper we showed that the pre-inflationary scenario proposed in [6] to resolve the overshoot problem of small-field inflation leads to the formation of giant spherically

symmetric overdense regions. The number of these giant overdense regions in the visible universe is exponentially sensitive to ΔN_e, and so cannot be determined with our present knowledge.

Since typically the structure in the universe is not formed in a spherically symmetric fashion this appears to be a distinct feature of [6] that cannot be confused with other possible imprints due to finite ΔN_e or higher order terms. Hence we believe that even a detection of a single giant spherically symmetric overdense region with a radius of at least 110 Mpc should be viewed as evidence for the scenario of [6]. What in our opinion should be viewed as a clear cut evidence for [6] is a detection of several spherically symmetric giant overdense regions with the same properties. The reason is that it seems extremely unlikely that a different scenario could lead to a similar anomaly in structure formation.

Since the properties of the giant structure are fixed by a specific parameter in the theory, $m_{,\phi}$, such a development could perhaps open an interesting dialog between cosmology and string theory.

Acknowledgements

A   Appendix

Here we derive (3.9). The starting point is (3.5). To solve this equation it is useful to define the Sasaki-Mukhanov variable [23]

$$v \equiv a\delta\phi \tag{A.1}$$

and to switch to conformal time, $d\tau = dt/a(t)$, which in de-Sitter space gives

$$\tau = -\frac{1}{Ha}. \tag{A.2}$$

In these variables (3.5) reads

$$\ddot{v} - \left(\nabla^2 + \frac{\ddot{a}}{a}\right)v = -m_{,\phi}\delta^3(x_i), \tag{A.3}$$

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which in momentum space gives

$$\ddot{v}_k + \left(k^2 - \frac{\ddot{a}}{a}\right)v_k = -\frac{m_{,\phi}}{(2\pi)^{3/2}}. \tag{A.4}$$

In de-Sitter space $\frac{\ddot{a}}{a} = \frac{2}{\tau^2}$ and the homogenous solutions $(m_{,\phi} = 0)$ take the form

$$v_k = A_1(k)F_1(k,\tau) + A_2(k)F_2(k,\tau), \tag{A.5}$$

where

$$F_1(k,\tau) = \frac{1}{\tau}\left(\cos(\tau k)\tau k - \sin(\tau k)\right), \quad F_2(k,\tau) = \frac{1}{\tau}\left(\cos(\tau k) + \sin(\tau k)\tau k\right). \tag{A.6}$$

These solutions have the well known behavior. At early times when $\tau k \gg 1$ they oscillate

$$F_1(k,\tau) = k\cos(\tau k), \quad F_2(k,\tau) = k\sin(\tau k), \tag{A.7}$$

and at late times $(\tau \to 0^-)$ they give a decaying mode and a growing mode (that becomes a constant when transforming back to the original $\delta\phi_k$)

$$F_1(k,\tau) = -\frac{1}{3}\tau^2 k^3, \quad F_2(k,\tau) = \frac{1}{\tau}. \tag{A.8}$$

The inhomogeneous solution of

$$\ddot{v}_k + \left(k^2 - \frac{2}{\tau^2}\right)v_k = g(t) \tag{A.9}$$

can be written in the following form

$$v_k(\tau) = \frac{1}{k^3}\left(F_2(k,\tau)\int^\tau F_1(k,\tilde\tau)g(\tilde\tau)d\tilde\tau - F_1(k,\tau)\int^\tau F_2(k,\tilde\tau)g(\tilde\tau)d\tilde\tau\right). \tag{A.10}$$

We are interested in the case $g(t) = -\frac{m_{,\phi}}{(2\pi)^{3/2}}$ in which

$$\int^\tau F_1(k,\tilde\tau)g(\tilde\tau)d\tilde\tau = -\frac{m_{,\phi}}{(2\pi)^{3/2}}\left(\sin(\tau k) - \mathrm{Si}(\tau k) + C_1\right), \tag{A.11}$$

$$\int^\tau F_2(k,\tilde\tau)g(\tilde\tau)d\tilde\tau = -\frac{m_{,\phi}}{(2\pi)^{3/2}}\left(-\cos(\tau k) + \mathrm{Ci}(\tau k) + C_2\right). \tag{A.12}$$

The constants of integration C_1 and C_2 are fixed by the initial condition (3.6) at $\tau \to -\infty$, which in terms of the variable v_k is

$$v_k = -\frac{m_{,\phi}}{(2\pi)^{3/2}k^2}. \tag{A.13}$$

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This implies that

$$C_1 = \text{Si}(-\infty) = -\frac{\pi}{2}, \qquad C_2 = -\text{Ci}(-\infty) = 0. \tag{A.14}$$

Therefore at late times ($\tau \to 0^-$) we find that

$$v_k = \frac{m_{,\phi}}{\sqrt{32\pi}\tau k^3}, \tag{A.15}$$

and that

$$\delta\phi_k = -\frac{m_{,\phi}H}{\sqrt{32\pi}k^3}. \tag{A.16}$$

References

[1] A. H. Guth, The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems, Phys. Rev. D **23**, 347 (1981).
A. D. Linde, A New Inflationary Universe Scenario: A Possible Solution Of The Horizon, Flatness, Homogeneity, Isotropy And Primordial Monopole Problems, Phys. Lett. B **108**, 389 (1982).
A. Albrecht and P. J. Steinhardt, Cosmology For Grand Unified Theories With Radiatively Induced Symmetry Breaking, Phys. Rev. Lett. **48**, 1220 (1982).

[2] E. Komatsu *et al.* [WMAP Collaboration], "Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations:Cosmological Interpretation," arXiv:0803.0547 [astro-ph].

[3] N. Kaloper, M. Kleban, A. E. Lawrence and S. Shenker, "Signatures of short distance physics in the cosmic microwave background," Phys. Rev. D **66**, 123510 (2002) [arXiv:hep-th/0201158].
C. P. Burgess, J. M. Cline and R. Holman, "Effective field theories and inflation," JCAP **0310**, 004 (2003) [arXiv:hep-th/0306079].
N. Kaloper and M. Kaplinghat, "Primeval corrections to the CMB anisotropies," Phys. Rev. D **68**, 123522 (2003) [arXiv:hep-th/0307016].
M. Porrati, "Bounds on generic high-energy physics modifications to the primordial power spectrum from back-reaction on the metric," Phys. Lett. B **596**, 306 (2004) [arXiv:hep-th/0402038].

B. R. Greene, K. Schalm, G. Shiu and J. P. van der Schaar, "Decoupling in an expanding universe: Backreaction barely constrains short distance effects in the CMB," JCAP **0502**, 001 (2005) [arXiv:hep-th/0411217].

B. A. Powell and W. H. Kinney, "The pre-inflationary vacuum in the cosmic microwave background," Phys. Rev. D **76**, 063512 (2007) [arXiv:astro-ph/0612006].

S. Chang, M. Kleban and T. S. Levi, "When Worlds Collide," JCAP **0804**, 034 (2008) [arXiv:0712.2261 [hep-th]].

[4] R. Brustein and P. J. Steinhardt, "Challenges for superstring cosmology," Phys. Lett. B **302**, 196 (1993) [arXiv:hep-th/9212049].

[5] N. Kaloper and K. A. Olive, "Dilatons in string cosmology," Astropart. Phys. **1**, 185 (1993).

A. A. Tseytlin and C. Vafa, "Elements Of String Cosmology," Nucl. Phys. B **372**, 443 (1992) [arXiv:hep-th/9109048].

A. A. Tseytlin, "String Cosmology And Dilaton," arXiv:hep-th/9206067.

T. Barreiro, B. de Carlos and E. J. Copeland, "Stabilizing the dilaton in superstring cosmology," Phys. Rev. D **58**, 083513 (1998) [arXiv:hep-th/9805005].

M. Dine, "Towards a solution of the moduli problems of string cosmology," Phys. Lett. B **482**, 213 (2000) [arXiv:hep-th/0002047].

G. Huey, P. J. Steinhardt, B. A. Ovrut and D. Waldram, "A cosmological mechanism for stabilizing moduli," Phys. Lett. B **476**, 379 (2000) [arXiv:hep-th/0001112].

R. Brustein, S. P. de Alwis and P. Martens, "Cosmological stabilization of moduli with steep potentials," Phys. Rev. D **70**, 126012 (2004) [arXiv:hep-th/0408160].

B. Freivogel, M. Kleban, M. Rodriguez Martinez and L. Susskind, "Observational consequences of a landscape," JHEP **0603**, 039 (2006) [arXiv:hep-th/0505232].

[6] N. Itzhaki and E. D. Kovetz, "Inflection Point Inflation and Time Dependent Potentials in String Theory," JHEP **0710**, 054 (2007) [arXiv:0708.2798 [hep-th]].

[7] N. Kaloper, J. Rahmfeld and L. Sorbo, "Moduli entrapment with primordial black holes," Phys. Lett. B **606**, 234 (2005) [arXiv:hep-th/0409226].

A. Berndsen, T. Biswas and J. M. Cline, JCAP **0508**, 012 (2005) [arXiv:hep-th/0505151].

S. Cremonini and S. Watson, "Dilaton dynamics from production of tensionless membranes," Phys. Rev. D **73**, 086007 (2006) [arXiv:hep-th/0601082].

[8] E. Silverstein and A. Westphal, "Monodromy in the CMB: Gravity Waves and String Inflation," arXiv:0803.3085 [hep-th].

[9] S. H. Henry Tye, "Brane inflation: String theory viewed from the cosmos," Lect. Notes Phys. **737**, 949 (2008) [arXiv:hep-th/0610221].
J. M. Cline, "String cosmology," arXiv:hep-th/0612129.
R. Kallosh, "On Inflation in String Theory," Lect. Notes Phys. **738**, 119 (2008) [arXiv:hep-th/0702059].
C. P. Burgess, "Lectures on Cosmic Inflation and its Potential Stringy Realizations," PoS **P2GC**, 008 (2006) [Class. Quant. Grav. **24**, S795 (2007)] [arXiv:0708.2865 [hep-th]].
L. McAllister and E. Silverstein, "String Cosmology: A Review," Gen. Rel. Grav. **40**, 565 (2008) [arXiv:0710.2951 [hep-th]].

[10] R. Brandenberger, G. Geshnizjani and S. Watson, "On the initial conditions for brane inflation," Phys. Rev. D **67**, 123510 (2003) [arXiv:hep-th/0302222].

[11] P. Binetruy and M. K. Gaillard, Candidates For The Inflaton Field In Superstring Models, Phys. Rev. D**34** (1986) 3069.
T. Banks, M. Berkooz, S. H. Shenker, G. W. Moore and P. J. Steinhardt, Modular Cosmology, Phys. Rev. D**52**, 3548 (1995) [arXiv:hep-th/9503114

[12] A. H. Guth and S. Y. Pi, "Fluctuations In The New Inflationary Universe," Phys. Rev. Lett. **49** (1982) 1110.

[13] A. R. Liddle and D. H. Lyth, "Cosmological inflation and large-scale structure," *Cambridge, UK: Univ. Pr. (2000)*

[14] F. Hoyle and M. S. Vogeley, "Voids in the 2dF Galaxy Redshift Survey," Astrophys. J. **607**, 751 (2004) [arXiv:astro-ph/0312533].

[15] S. G. Patiri, F. Prada, J. Holtzman, A. Klypin and J. Betancort-Rijo, "The Properties of Galaxies in Voids," Mon. Not. Roy. Astron. Soc. **372**, 1710 (2006) [arXiv:astro-ph/0605703].

[16] R. Allahverdi, K. Enqvist, J. Garcia-Bellido, A. Jokinen and A. Mazumdar, MSSM flat direction inflation: slow roll, stability, fine tuning and reheating, JCAP **0706**, 019 (2007) [arXiv:hep-ph/0610134]
A. Mazumdar, New developments on embedding inflation in gauge theory and particle physics, arXiv:0707.3350 [hep-ph].
G. G. Ross and S. Sarkar, Successful supersymmetric inflation, Nucl. Phys. B **461** (1996) 597 [arXiv:hep-ph/9506283].
G. German, G. G. Ross and S. Sarkar, Low-scale inflation, Nucl. Phys. B **608** (2001) 423 [arXiv:hep-ph/0103243].

[17] L. Kofman, A. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, "Beauty is attractive: Moduli trapping at enhanced symmetry points," JHEP **0405**, 030 (2004) [arXiv:hep-th/0403001].

[18] N. Afshordi, M. Zaldarriaga and K. Kohri, "On the stability of dark energy with mass-varying neutrinos," Phys. Rev. D **72**, 065024 (2005) [arXiv:astro-ph/0506663].

[19] R. Fardon, A. E. Nelson and N. Weiner, "Dark energy from mass varying neutrinos," JCAP **0410**, 005 (2004) [arXiv:astro-ph/0309800].

[20] P. Vielva, E. Martinez-Gonzalez, R. B. Barreiro, J. L. Sanz and L. Cayon, "Detection of non-Gaussianity in the WMAP 1-year data using spherical wavelets," Astrophys. J. **609**, 22 (2004) [arXiv:astro-ph/0310273].
M. Cruz, M. Tucci, E. Martinez-Gonzalez and P. Vielva, "The non-Gaussian Cold Spot in WMAP: significance, morphology and foreground contribution," Mon. Not. Roy. Astron. Soc. **369**, 57 (2006) [arXiv:astro-ph/0601427].
M. Cruz, L. Cayon, E. Martinez-Gonzalez, P. Vielva and J. Jin, "The non-Gaussian Cold Spot in the 3-year WMAP data," Astrophys. J. **655**, 11 (2007) [arXiv:astro-ph/0603859].

[21] K. T. Inoue and J. Silk, "Local Voids as the Origin of Large-angle Cosmic Microwave Background Anomalies: The Effect of a Cosmological Constant," Astrophys. J. **664**, 650 (2007) [arXiv:astro-ph/0612347].
K. T. Inoue and J. Silk, "Local Voids as the Origin of Large-angle Cosmic Microwave Background Anomalies," Astrophys. J. **648**, 23 (2006) [arXiv:astro-ph/0602478].

L. Rudnick, S. Brown and L. R. Williams, "Extragalactic Radio Sources and the WMAP Cold Spot," arXiv:0704.0908 [astro-ph].

[22] K. M. Smith and D. Huterer, "No evidence for the cold spot in the NVSS radio survey," arXiv:0805.2751 [astro-ph].

[23] M. Sasaki, "Large Scale Quantum Fluctuations in the Inflationary Universe," Prog. Theor. Phys. **76**, 1036 (1986).
V. F. Mukhanov, "Quantum Theory of Gauge Invariant Cosmological Perturbations," Sov. Phys. JETP **67**, 1297 (1988) [Zh. Eksp. Teor. Fiz. **94N7**, 1 (1988)].